

<http://tinyurl.com/k9alzr3>

Chapter 1

Slope and Rate of Change

1.1 Rise Over Run

1.2 Grade, Angle of Elevation, and Distance

1.3 Rate of Change



Name: _____

1.1 Rise Over Run

Working with Ratio and Proportion

What is a **ratio**? What do you think about when you hear it?

- A ratio is a relationship between two measurable quantities or things.
- For example: distance per time (speed), height per horizontal distance, gas mileage (kilometers per liter).

Example

A recipe for vegetarian chili contains:

- 56 oz of chopped tomatoes
- 30 oz of kidney beans
- 15 oz of whole kernel corn

a) What is the ratio of corn to kidney beans?

b) What is the ratio of tomatoes to kidney beans?

A ratio can have a numerator larger than the denominator. Because a ratio compares two numbers, *do not* rewrite it as a mixed fraction.

A **proportion** is a statement of equality between two ratios.

Example

A big-screen TV has an aspect ratio of 16:9, which means that for every 16 inches of width, the TV is 9 inches high. What is the height of a TV that is 40 inches wide?

Worksheet: Ratio and Proportion

Ratio and Proportion Worksheet

1. In a juice mixture, 750 mL of water are mixed with 250 mL of juice concentrate. What is the ratio of concentrate to water?

2. To mix a certain shade of green paint, a painter mixes 2.3 L of blue paint with 1.7 L of yellow paint. What is the ratio of blue paint to yellow paint? Express your answer as a fraction.

3. A cereal mixture contains 6 cups of oats, 2 cups of almonds, 1 cup of raisins, and $\frac{3}{4}$ cup of coconut.
 - a) What is the ratio of oats to raisins?

 - b) What is the ratio of almonds to coconut?

 - c) What is the ratio of oats to the total ingredients in the recipe?

4. A dirt bike requires 15 L of gas to be mixed with 4 L of oil. If you use 20 L of gas, how much oil will you need?

5. If 10 cm on a map represents 25 km of actual ground, how many centimetres would 45 km of actual ground be on the map?

6. A recipe for corn chowder includes 3 cups of corn, 2 cups of water, and $1\frac{1}{2}$ cups of cream. If you increase the yield of the recipe and use $4\frac{1}{2}$ cups of cream, how much corn will you need?

Answers:

1) 1:3 2) 23:17 3)a) 6:1 b) 8:3 c) 8:13 4) 5.3 L 5) 18cm 6) 9 cups

Working with Slope

You may have heard the words pitch, slant, or steepness. What do these terms mean?

They are words to describe **slope**. Slope is a ratio that compares the _____
_____.

It is a ratio between these two numbers.

Slope can be expressed as follows:

$$\text{slope} = \frac{\Delta \text{ vertical distance}}{\Delta \text{ horizontal distance}}$$

The variable m is used to represent slope. The change in vertical distance is also called the _____, and the change in horizontal distance is also called the _____.

$$m = \frac{\text{rise}}{\text{run}}$$

Note: Slope does not have units because it is a ratio, not a measurement.

Example

1. Calculate the slope as a fraction in the simplest form and as a decimal.

<i>Rise</i>	<i>Run</i>	<i>Slope</i>	
		<i>As a fraction</i> $\left(m = \frac{\text{rise}}{\text{run}}\right)$	<i>As a decimal</i>
18 m	63 m		
21 m	49 m		
1.2 cm	0.6 cm		
12.4 mm	4.6 mm		
300 ft	900 ft		

2.

a) Calculate the slope of a line that has a rise of 12 cm for a run of 8 cm.

b) The slope of a line is $\frac{7}{20}$. What is the rise if the run is 100 metres?

c) Harry is building a ramp in two sections, both with the same slope. If one section rises 2 m for a run of 6.5 m, how much will it have to rise for the remaining run of 9.8 m?

3. Find the slope of the following triangles

a) If the rise of the cookbook stand is 15cm, and the run is 10cm long, what is the slope of the cookbook stand?



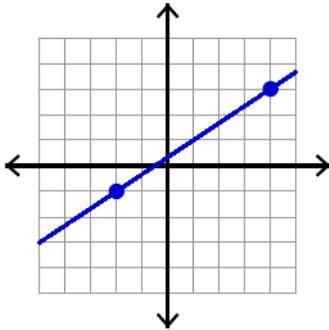
b) You lay a loading ramp against a truck that is 18 inches high. The slope of the ramp is 0.2. What is the horizontal distance?



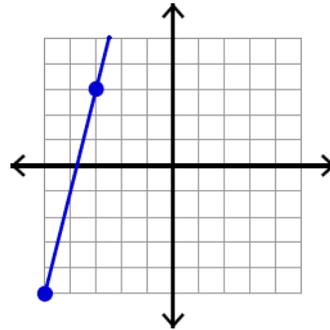
Using the Rise and Run determine the slope of each line.

Find the slope of each line.

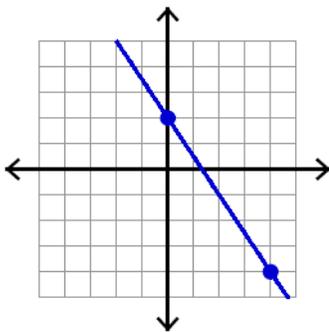
1)



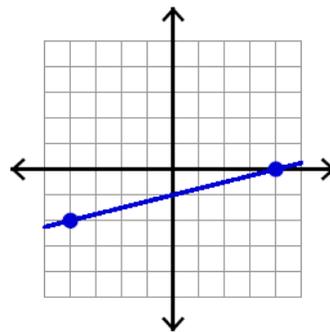
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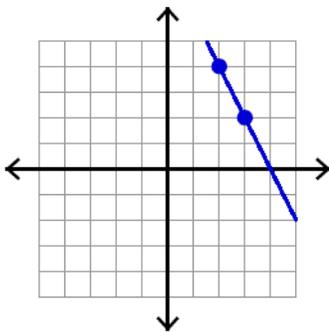
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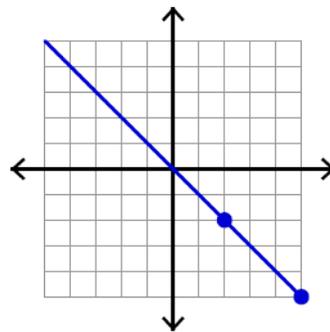
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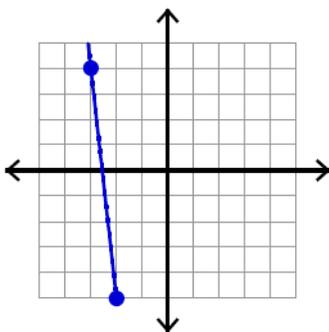
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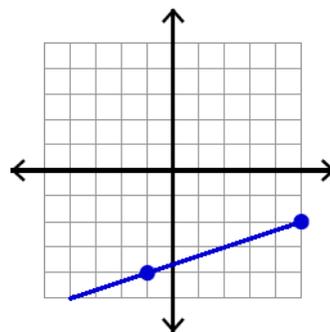
6)



7)



8)



Activity 1.1: Creating and Measuring Slope

In this activity you will work with a partner to create a series of slopes and describe your observations.

Remove the pencil after each stage.

Materials:

- a hard cover textbook
- pencil or pen that will roll

1) Place a textbook on a flat surface. Place a pencil on the book. What do you observe?

2) Put a thumb under the cover of the textbook and put the pencil on it. What did you observe?

a) Why did this happen?

b) What changed from trial 1?

c) What stayed the same?

3) Put a fist under the cover of the textbook and put the pencil on it. What did you observe?

d) Why did this happen?

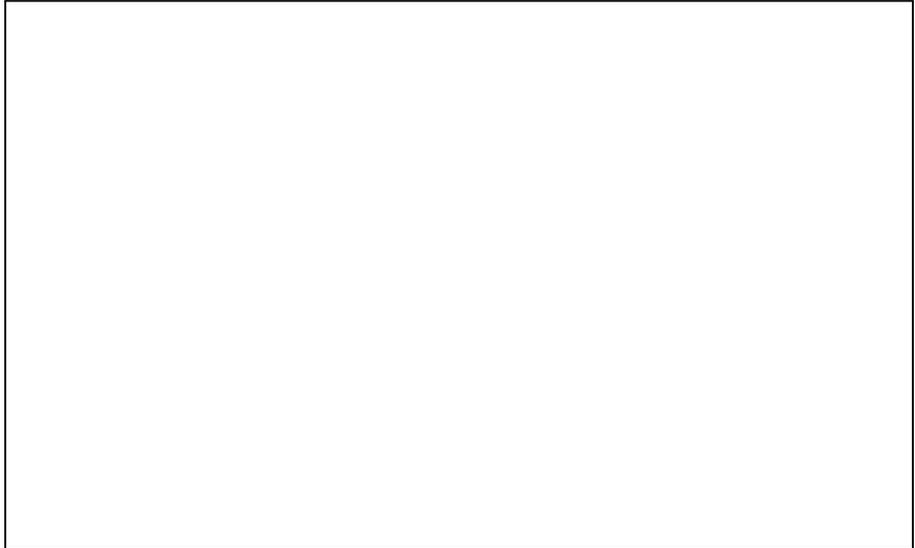
e) What changed from trial 1?

f) What stayed the same?

- 4) With a ruler have one partner measure the rise and run. Draw the right triangle and label it below.

$$m = \frac{\text{rise}}{\text{run}}$$

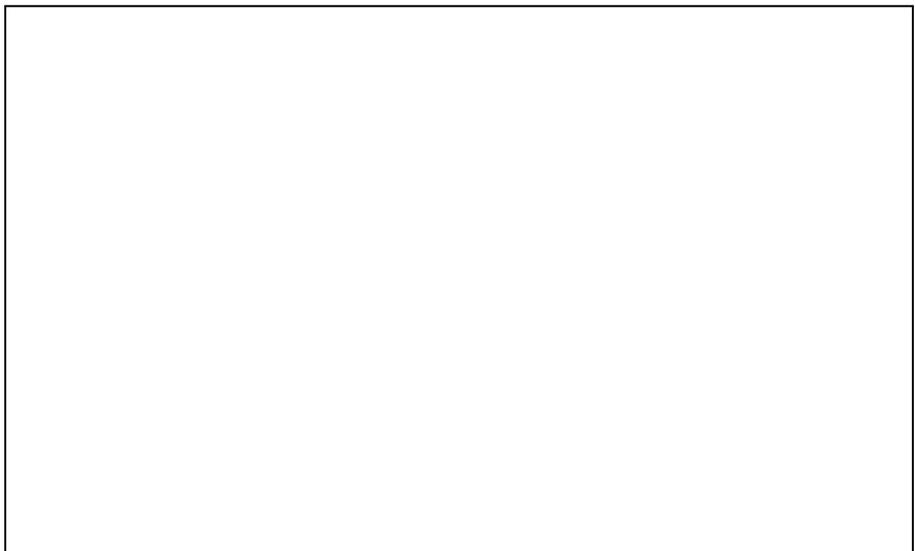
What is the slope?



- 5) Now have your other partner put their fist under the textbook cover. Draw the right triangle and label it below.

$$m = \frac{\text{rise}}{\text{run}}$$

What is the slope?



Did you have the same slopes? Explain why or why not?

Slope Worksheet

1. Calculate the slope in the following situations.
 - a) A wheelchair ramp has a rise of 3 feet and a run of 18 feet.
 - b) A snowboard jump rises 1.25 m over 5 m of horizontal distance.
 - c) A roof rises 8 feet over a horizontal distance of 18 feet.
 - d) A hill rises 10 metres over a horizontal distance of 8 metres.
 - e) A slide covers 3.5 m of ground and is 2.4 m tall.
2. The slope of a hill is $\frac{3}{190}$. The hill has a rise of 400 m. What is the horizontal distance covered by the hill?
3. The slope of a staircase is 0.95. If it rises 210 cm, what is the run?
4. The slope of a street is 0.54. If it covers 28 m of horizontal distance, what is the rise of the street?

5. Leslie works for a shipping company. He regularly carries boxes up and down several stairs and has decided that it would be easier if he built a ramp. The stairs have a rise of 3.5 m for a run of 6.0 m. What is the slope of the stairs?
6. Harry is building a staircase with a slope of 0.89. If the total rise of the staircase is 203 cm, what is the total run of the stairway?
7. The slope of a slide for a playground is to be $\frac{17}{10}$. If the maximum space available for the slide is a horizontal distance of 1.5 m, how high will the slide be?
8. Use the information given to complete the table.

<i>Rise</i>	<i>Run</i>	<i>Slope</i>
15 ft		$\frac{1}{4}$
12 cm		0.375
	16 m	$\frac{9}{5}$
	42 in	$\frac{32}{7}$
63 m		3.0
19.5 ft		0.25

Answers:

- 1)a) $\frac{1}{6}$ b) $\frac{1}{4}$ c) $\frac{4}{9}$ d) $\frac{5}{4}$ e) $\frac{24}{35}$ 2) 25333m 3) 221.05cm 4) 15.12 5) 0.583 6) 180.67cm 7) 2.55 8) 60; 32; 28.8m; 192in; 21; 78

1.2 Grade, Angle of Elevation, and Distance

Has anyone every seen a sign like this on the road? Where would you find it? What is it warning drivers about? Which drivers need to pay attention?

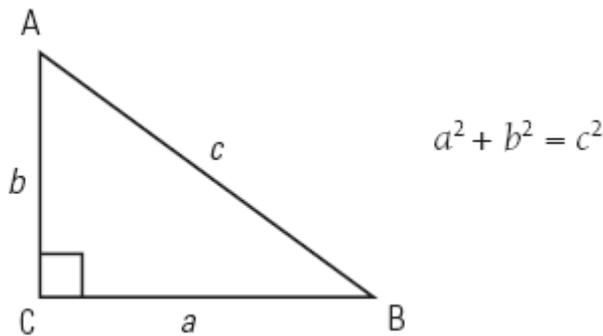


What does the “20%” on the sign represent? In this section we’ll find out.

- The slope of a road or other surface is called a _____.
- A _____. Does anyone remember how we get a percentage, or what the percentage (%) sign means? It has to do with 100, and division.
- To convert slope to percent grade, just multiply the value you get for slope by 100 (initially we have our slope in decimal form).

Review: The Pythagorean Theorem and the Tangent Ratio

The Pythagorean Theorem states the relationship between the sides of a _____. In right triangle $\triangle ABC$ shown below, the Pythagorean Theorem states the following:



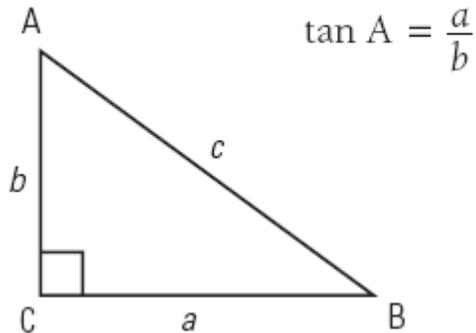
The tangent ratio is a trigonometric ratio that also applies to right triangles. It is a ratio of the side opposite an acute angle to the side adjacent to the angle. For angle A, the ratio can be stated as follows:

$$\text{tangent } \angle A = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$$

And is abbreviated:

$$\tan A = \frac{\text{opp}}{\text{adj}}$$

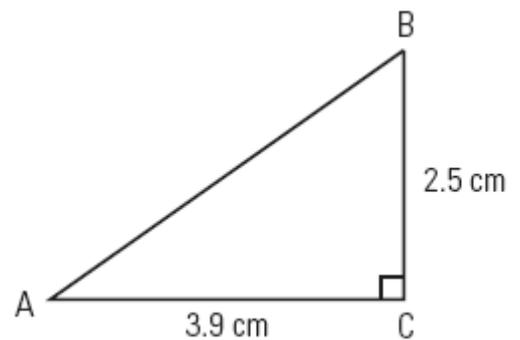
And for $\triangle ABC$:



Example

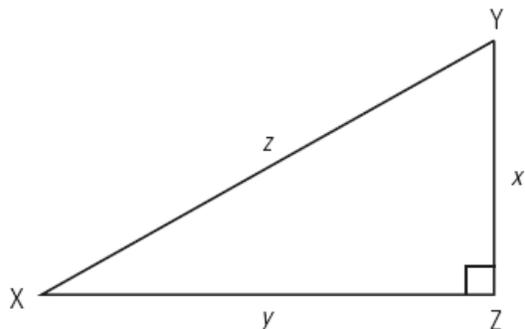
Use the diagram to calculate the following:

- the length of the hypotenuse
- the measure of angle A.



Working with Slope and the Tangent Ratio

In the right triangle $\triangle XYZ$, the slope of the segment z is the ratio of the rise, x , to the run, y .



$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{x}{y}$$

The tangent of angle X is the ratio of the opposite side, x, to the adjacent side, y.

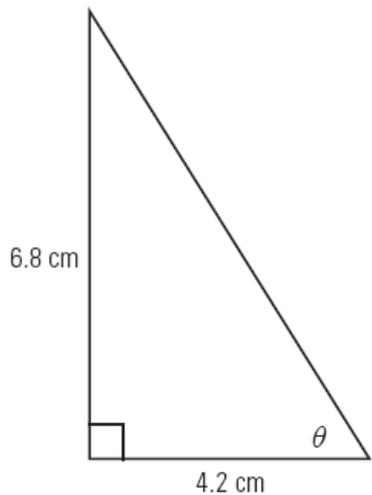
$$\tan X = \frac{\text{opp}}{\text{adj}}$$

$$\tan X = \frac{x}{y}$$

Thus, the slope of segment z, the hypotenuse of the triangle, is the same as the tangent of angle X .

Angle X can be referred to as the angle of _____ of segment z. Angle Y is called its angle of _____.

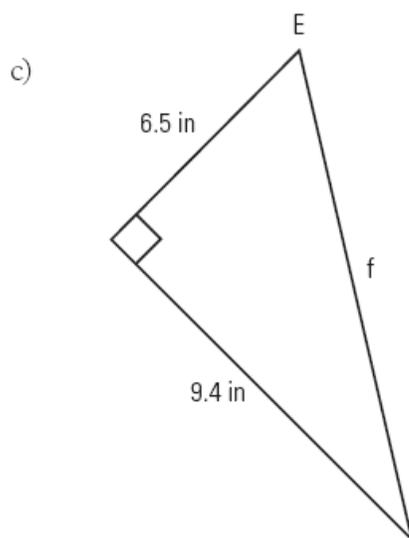
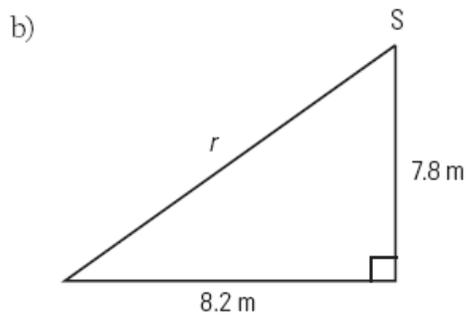
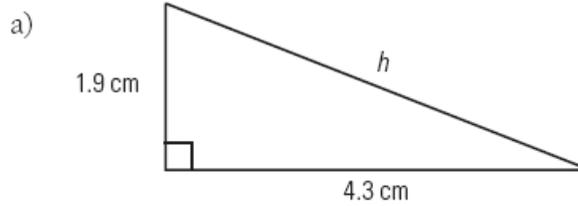
Example



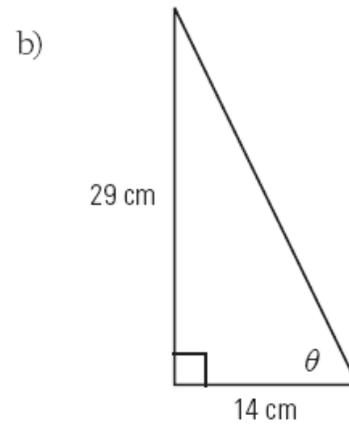
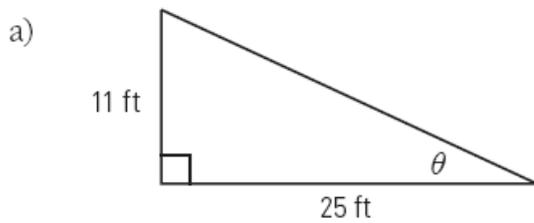
Find the slope of the hypotenuse as a fraction, and use it to find the angle of elevation.

Pythagorean Theorem and Tangent Ratio with Slope

1. In each diagram, calculate the indicated side using the Pythagorean Theorem and find the indicated angle using the tangent ratio.



2. Calculate the angle of elevation and the slope of the hypotenuse.



3. A ski jump rises 3 feet over a run of 7 feet.

a) What is the length of the surface of the jump?

b) What is the angle of elevation of the jump?

4. A driveway rises 2.2 m from the street level to the carport, which is a horizontal distance of 5.8 m from the street.

a) How long is the driveway?

b) What is its angle of elevation?

5. A wheelchair ramp is being built to rise to a landing that is 2.4 m above the ground. Building regulations say that the ramp can have a maximum rise of 2.5 cm per 30 cm of run.

a) What is the total run for the wheelchair ramp?

b) How long will the ramp be?

Answers:

1) a) 4.7 cm; 23.8° b) 11.3 m; 46.4° c) 11.4 in; 55.3° 2) a) 23.7°; 0.44 b) 64.2°; 2.07 3) a) 7.6 ft b) 23.2° 4) a) 6.2 m b) 20.8° 5) a) 28.8 m b) 28.9 m

Working with Grade and Pitch

Grade

When talking about the slope of a road, it is usually called the road grade. Grade is commonly expressed as a percentage. The steeper the road, the higher its percent grade.

The following formula can be used to calculate grade.

$$\text{percent grade} = \frac{\text{rise}}{\text{run}} \times 100$$



Road grades must be kept to a minimum because a steeper grade limits the size of load that can be hauled. A 1% grade halves the load that can be hauled!

Example

Martin is driving his truck on the Trans-Canada Highway. There is a sign indicating a 7% grade.

a) What is the slope of the road, as a fraction?

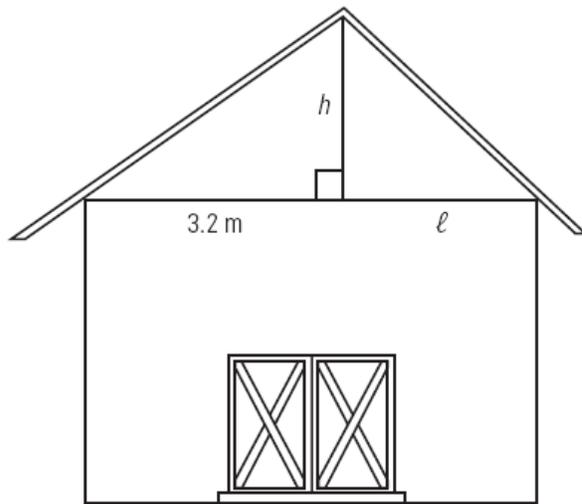
b) Find the angle of elevation of the road.

Pitch

The slope of a roof is often referred to as its _____. A roof with a pitch of 5:8 is a roof with a rise of 5 and a run of 8, or a slope of $\frac{5}{8}$.

Example

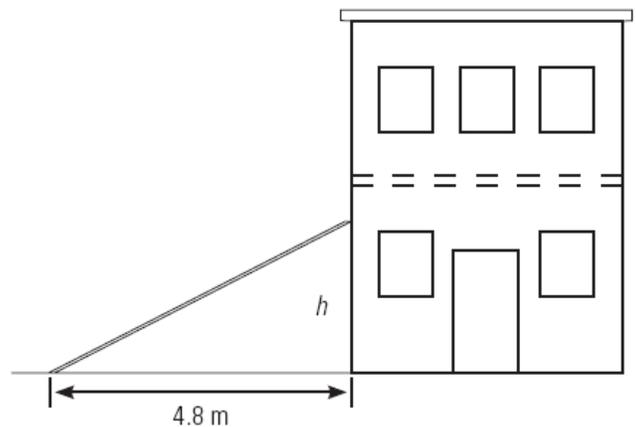
1. How wide is the shed in the diagram below if the pitch of the left side is 3:4 and the pitch of the right side is 8:9?



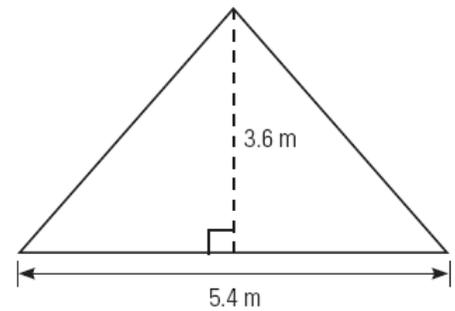
2. The roof of a tool shed has a pitch of 2:5. If the shed is 7 feet wide, what is the rise of the roof?

Working with Grade and Pitch Worksheet

1. Heckman Pass is a very steep section of highway connecting Anahim Lake and Bella Coola, BC. At its steepest section, it rises **900 m** over a run of **5 km**. What is the percent grade of this section?
2. So that water will drain properly, a patio attached to a house should slope downward about $2\frac{1}{2}$ inches for every 10 feet of run. Calculate:
 - a) the slope
 - b) the percent grade
3. One of the steepest railways in the world is the Lisbon tram in Portugal. In one section, it has a grade of 13.5%.
 - a) Express this as a slope.
 - b) Calculate the angle of elevation.
 - c) What is the rise for a run of 15 m?
4. The pitch of a roof on a lean-to that Dianna is building is 2:5. If the lean-to touches the ground 4.8 m from the base of the building, how high up the building does it reach?



5. What is the pitch of the roof of an A-frame building if its height is 3.6 m and its width is 5.4 m?



6. Alex works as a ski instructor. He has read that human-triggered avalanches occur most often on slopes with angles of elevation between 35° and 45° .

a) What are the slopes of these two angles of elevation?

b) What are the percent grades?

7. A roof rises 2 ft for every 5 ft of run.

a) What is the pitch of the roof?

b) What is the slope of the roof, as a decimal?

c) What is the percent grade?

Answers:

1) 18% 2) a) -0.021 b) 2.1% 3) a) 0.135 b) 7.7° c) 2.03m 4) 1.92m 5) 4:3 6) a) 0.7 and 1 b) 70% and 100% 7) a) 2:5 b) 0.4 c) 40%

1.3 Rate of Change

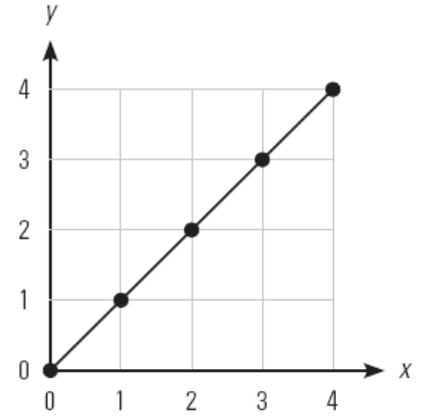
Using Coordinates to Calculate Slope

Another way of calculating slope is by using two points on a line. Choose any two points, (x_1, y_1) and (x_2, y_2) and use the formula:

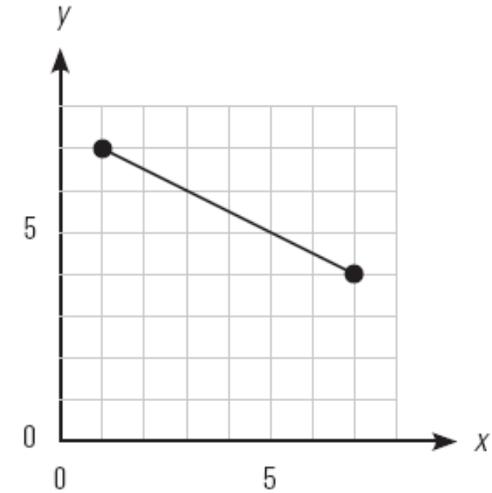
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

If a line on a graph rises from left to right, as in the following graph to the right, the slope is

_____.

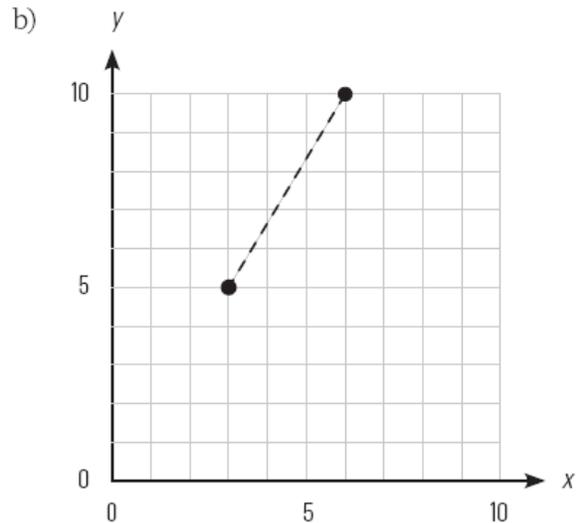
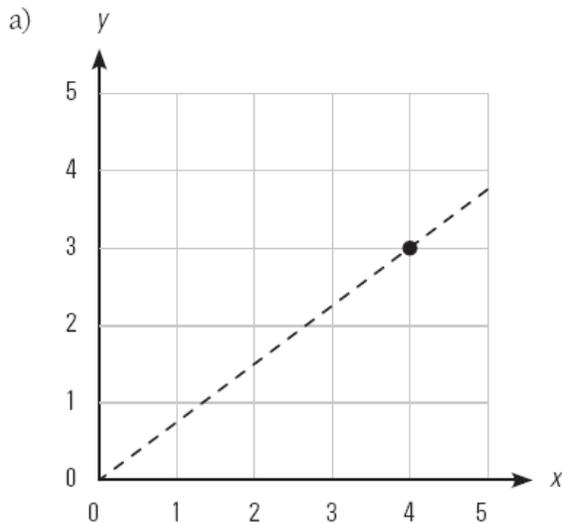


If a graph falls or goes down from left to right, the slope is _____ because the “rise” is downward, or in the negative direction. This graph has a negative slope.



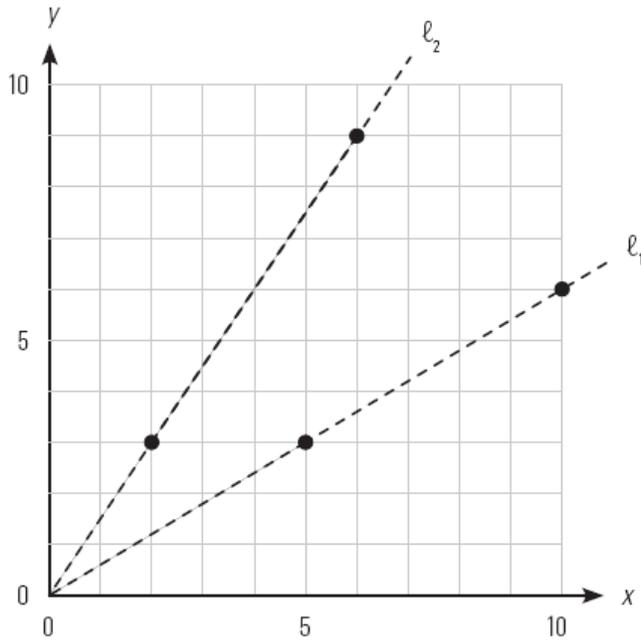
Example

Consider each of the following graphs. Are the slopes positive or negative? Calculate each slope, as a fraction.

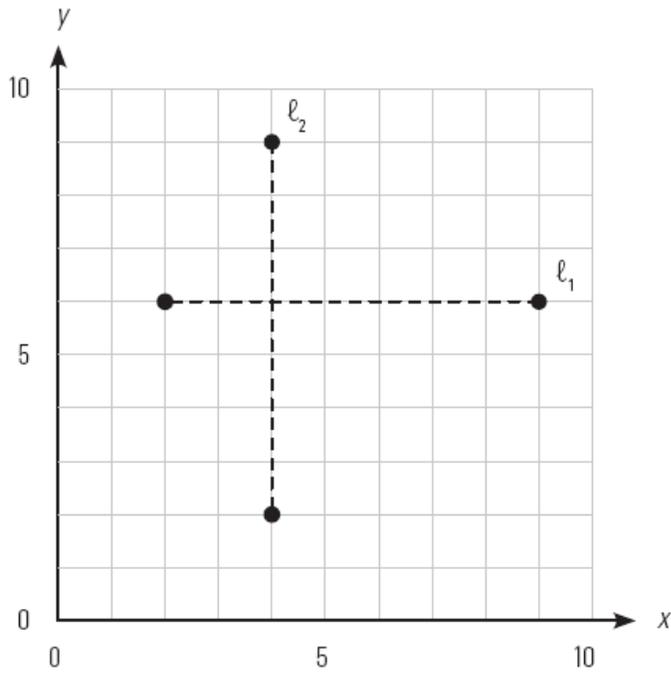


Rate of Change Worksheet

1. Calculate the slope of the two lines on the graph. Which is steeper?



2. Calculate the slopes of the two lines on the graph.



3. The line on a graph has the following points. Calculate the slope of the line.

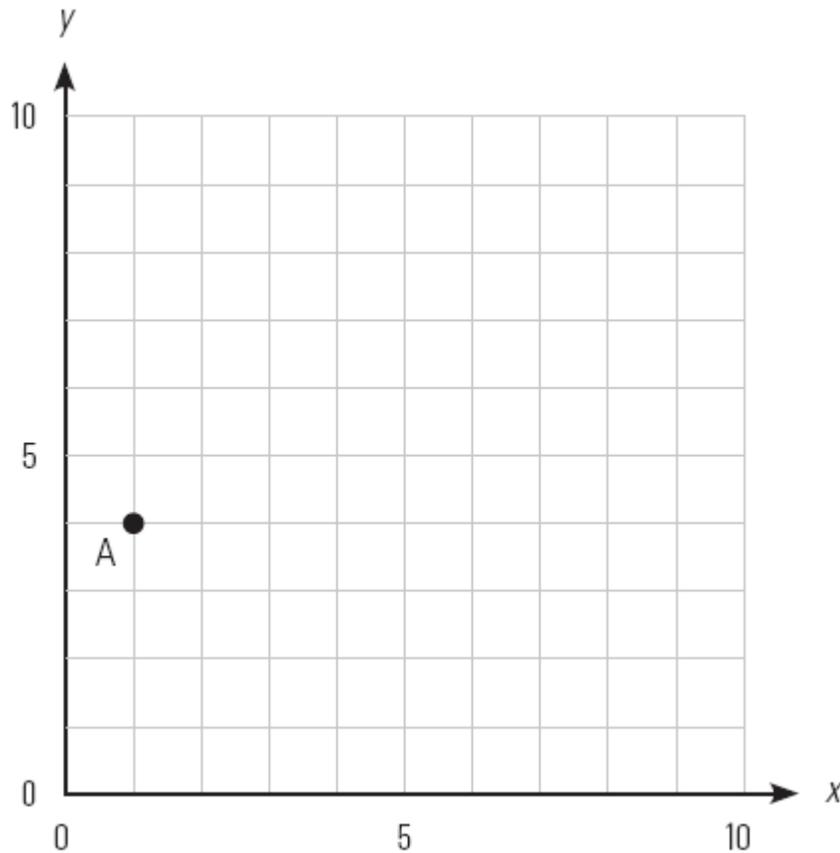
a) $(0, 0)$ and $(4, 204)$

b) $(12, 3)$ and $(16, 4.5)$

4. On the graph below, draw:

a) a solid line that passes through point A and has a slope of $\frac{6}{5}$

b) a dotted line that passes through point A and has a slope of $-\frac{3}{2}$.



Answers:

1) 0.6; 1.5; l_2 is steeper 2) zero; undefined 3) a) 51 b) $3/8$ 4) see teacher

Working with Slope as Rate of Change

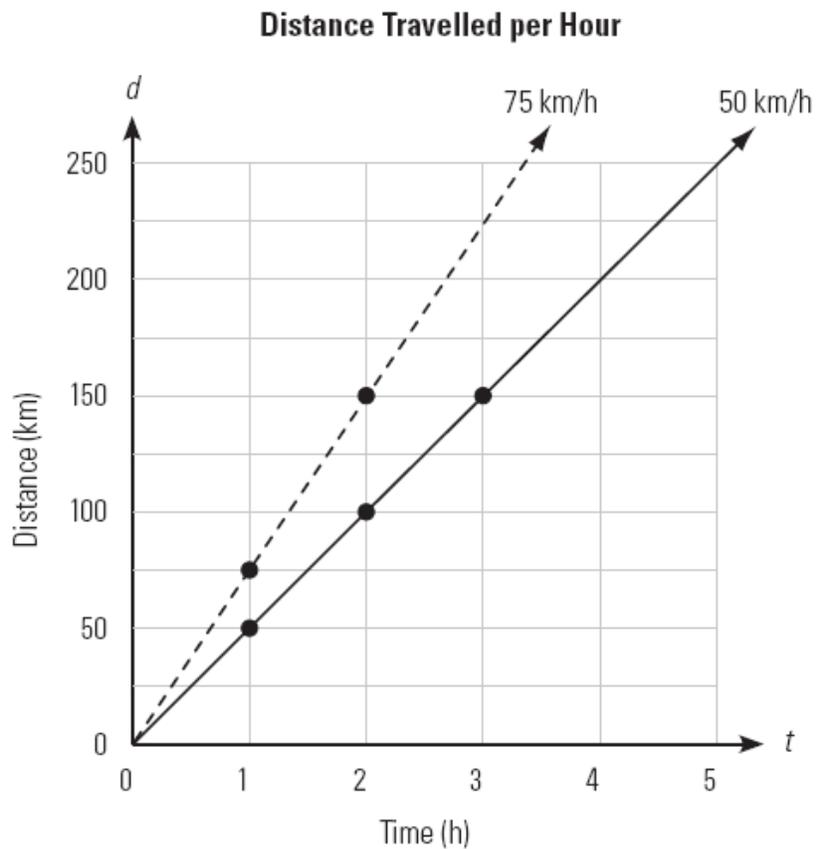
Graphs tell us visually the rate at which one variable changes compared to another variable.

Usually we have two variables. The two variables are called the **dependant variable** and the other is called the **independent variable**.

Dependant variable: This variable relies on the other variable.

Independent variable: A variable whose values will happen regardless.
(Usually time)

In the graph below, the distance we travel depends on how much time we spend moving. Time moves forward no matter how far you travel.



You can choose any two points and calculate the slope of the line.

For the dotted line:

$$m = \frac{150 - 75}{2 - 1} = \frac{75}{1} = 75 \text{ km/h}$$

For the solid line:

$$m = \frac{150 - 100}{2 - 1} = \frac{50}{1} = 50 \text{ km/h}$$

Each line has an equation that represents the relationship between distance and time:

$$d_{\text{dashed}} = 75t$$

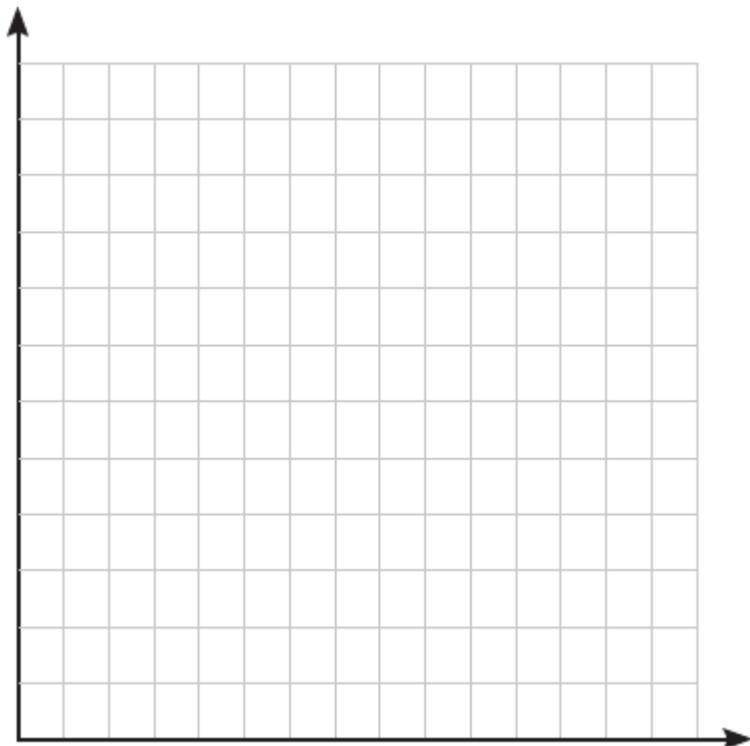
$$d_{\text{solid}} = 50t$$

Example

1. Willard works as an electrician's assistant and earns \$12.25 per hour.

- a) What is the dependent variable?
Write an equation that shows the relationship between hours worked and income.

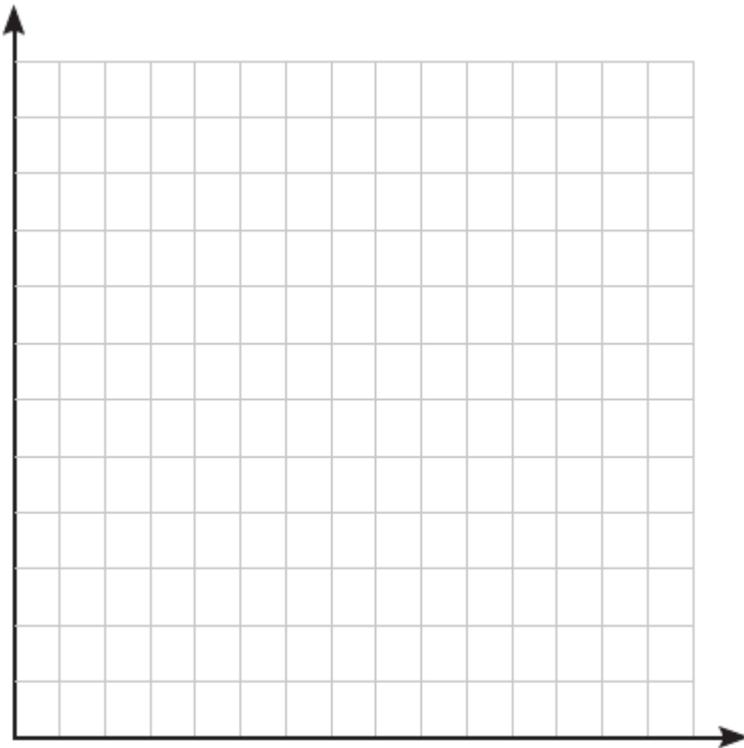
- b) Graph the equation.



- c) What is the slope of the graph and what does it represent?
- d) How much will Willard earn in 5 hours?
- e) If he earned \$183.75 on a job, how many hours did he work?

2. Reg timed himself jogging on a race course. He passed the 300-m point 1.5 minutes after starting and the 600-m point after 2.8 minutes.

- a) Show this on a graph.
- b) What was his average rate in m/min for this portion of the race?



Slope and Rate of Change Worksheet

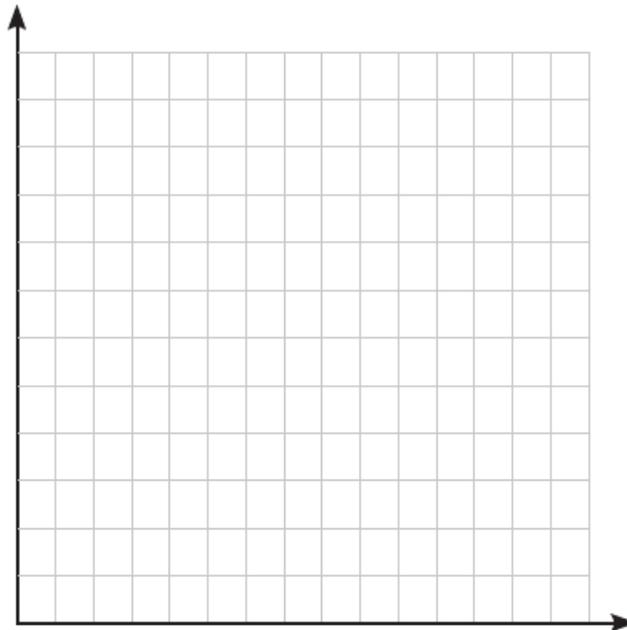
1. Reggie is starting to practice for a marathon.
 - a) If he walked 3 km in 30 minutes, what is his average rate in km/h?

 - b) Which is the independent variable?

 - c) If he does not change his pace, how long will it take him to walk a 36-km marathon?

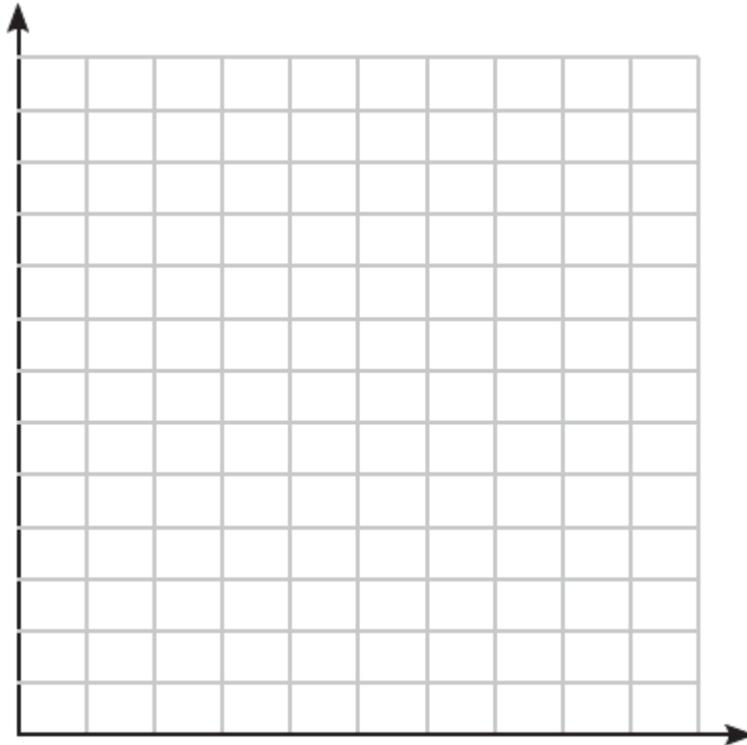
2. Jenita's car needs repairing, and she borrows \$600.00 from her brother to pay the bill. She repays her brother \$40.00 a week.
 - a) Write an equation that shows the relationship between how much she owes her brother and how much she pays each week.

 - b) Graph the information. What is the slope of the line and what does it represent?



3. Henrik earns \$73.20 in 6 hours at his weekend job. Jeff earns \$55.50 in 5 hours.

- a) Calculate their earnings for five different lengths of time.
- b) On the same graph, draw a solid line indicating Henrik's earnings and a dotted line showing Jeff's earnings.



- c) Who makes more money after an 8-hour shift?
- d) Calculate Henrik's and Jeff's rate of earning.

Answers:

1) a) 6 km/h b) time c) 6h 2) a) money owed = $40t$ b) $m=40$ (how much paid back per week) 3) a)b) see teacher c) Henrik d) Henrik: \$12.20; Jeff: \$11.10